CI Examples

- The average annual amount that $n = 1593$ families paid for college was \$24,164. If population S.D. is \$8500, give a 95% C.I. interval for μ, the average amount a family pays for a college under-graduate

<u>solution</u>:

$\bar{x} = 24{,}164$

$\sigma = 8500$

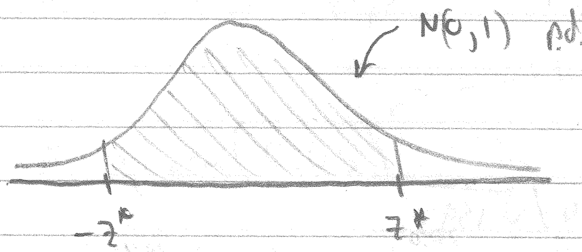
$n = 1593$

critical value

S.D. of $\bar{x}$

$$\bar{x} \pm \underbrace{z^* \, \sigma/\sqrt{n}}_{\text{Margin of error}}$$

$z_{0.05}$ = value s.t. 95% of mass is between $[-z^*, z^*]$



$\bar{x}$ will be within z^* S.D.'s of μ in 95% of repeated samples.

From 68, 95, 99.7 rule, $z^* = 1.96$ or ≈ 2

$\bar{x} \pm 2\sigma/\sqrt{n}$

$= 24{,}164 \pm 2 \cdot 8500/\sqrt{1593}$

$= (23{,}738,\ 24{,}590)$

- Interpretation: Using a procedure that captures the population average college spending in 95% of repeated samples, we say that average spending is between \$23,738 and \$24,590.

- Jargon: With 95% confidence, average spending is between \$23,738 and \$24,590.

- 68% confidence?

$\bar{x} \pm z^* \sigma/\sqrt{n}$

z^* now is s.t. 68% of mass lies between $[-z^*, z^*]$

From 68, 95, 99.7 rule, we have $z^* = 1$

$\bar{x} \pm \sigma/\sqrt{n}$

$= 24,164 \pm 8500/\sqrt{1593}$

$= (23,951,\ 24,377)$

- What do we notice about width of 95% and 68% C.I.'s?

95%: $24,590 - 23,738 = 852$
68%: $24,377 - 23,951 = 426$

A lot narrower!

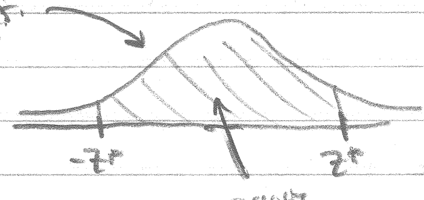
Intuition: If we don't need to capture μ as often, we can have a smaller interval

$\updownarrow$ "a larger net captures more fish"

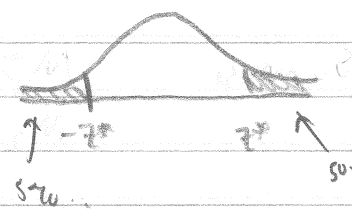
- General CI,

90% CI, Need to find z^* s.t.

N(0,1) p.d.f.

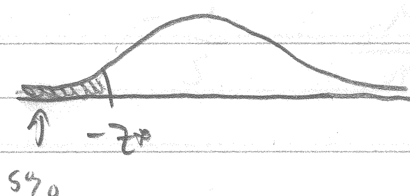


$= 1 -$



By p.d.f. property

$= 1 - 2 \cdot$



By symmetry property of Normal

$\Rightarrow -z^* = \text{qnorm}(0.05)$ in R $\quad = -1.645$

or use Normal Probability table

second digit of z

z	...	0.06	0.05	0.04	0.03	...
-1.7		- -	- -	- -	- -	
-1.6			0.0495	0.0505		
-1.5		- -	- -	- -	- -	

- mathematically:

find z^* s.t. $\Pr(-z^* \leq Z \leq z^*) = 0.9$ for $Z \sim N(0,1)$

$\Rightarrow 1 - \Pr(Z \leq -z^* \text{ or } Z > z^*) = 0.9$

$\Rightarrow 1 - 2 \cdot \Pr(Z \leq -z^*) = 0.9$

$\Rightarrow \Pr(Z \leq -z^*) = 0.05$

apply inverse CDF $\quad -z^* = -1.645$

- <u>Changing Sample Size</u>

M.O.E. = $z^* \sigma / \sqrt{n}$

- What if we want to decrease MOE by a factor of 2?

- we can decrease the confidence level (change z^*) or collect more samples (change n).

- Suppose $M_1 = z^* \sigma / \sqrt{n_1}$
  $M_2 = z^* \sigma / \sqrt{n_2}$

we want $\frac{M_2}{M_1} = \frac{1}{2}$

Find $\frac{n_1}{n_2}$

$$\frac{1}{2} = \frac{M_2}{M_1} = \frac{\cancel{z^*} \cancel{\sigma} / \sqrt{n_2}}{\cancel{z^*} \cancel{\sigma} / \sqrt{n_1}} = \frac{\sqrt{n_1}}{\sqrt{n_2}}$$

$\Rightarrow \frac{1}{4} = \frac{n_1}{n_2}$

$\Rightarrow n_2 = 4 n_1$

- So to halve the M.O.E., we need to quadruple the sample size.

- In general to decrease M.O.E. by a factor of $\gamma \in [0,1]$ we need to multiply sample size by $\frac{1}{\gamma^2}$