

## 02 Randomization Distribution

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# Learning Objectives

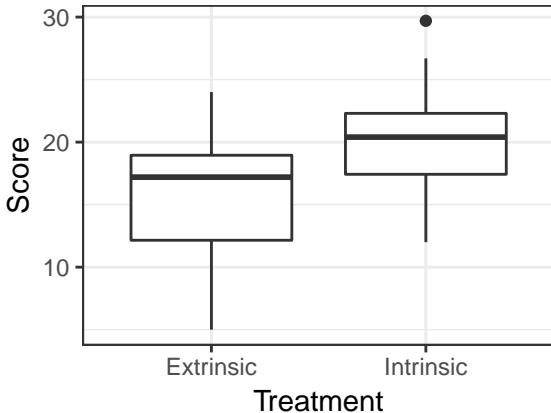
- Understand quantifying uncertainty using the randomization distribution.
- Section 1.3 in *Statistical Sleuth*.

## Case Study 1.1.1

```
library(Sleuth3)
data("case0101")
head(case0101)
```

```
##   Score Treatment
## 1    5.0 Extrinsic
## 2    5.4 Extrinsic
## 3    6.1 Extrinsic
## 4   10.9 Extrinsic
## 5   11.8 Extrinsic
## 6   12.0 Extrinsic
```

```
library(ggplot2)
qplot(x = Treatment, y = Score,
      data = case0101, geom = "boxplot")
```



```
ext_scores <- case0101$Score[case0101$Treatment == "Extrinsic"]
int_scores <- case0101$Score[case0101$Treatment == "Intrinsic"]
ext_mean <- mean(ext_scores)
int_mean <- mean(int_scores)
int_mean

## [1] 19.88

ext_mean

## [1] 15.74

int_mean - ext_mean

## [1] 4.144
```

## Question

Is 4.144 a big difference? A small difference?

Two possibilities

1.  $H_A$ : There is actually a difference in creativity scores between the two groups.
2.  $H_0$ : There is no difference, and 4.144 happened because, by *chance*, the intrinsic group happened to have more creative people in it.

We can explore how likely a value of 4.144 is if there were no difference.

# Hypothesize

- Let's suppose that there is no difference ( $H_0$ ) and that the people were going to get the same creativity score no matter which treatment they received.
- Would we expect the difference between groups to be exactly 0?

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- Would we expect the difference between groups to be exactly 0? (hint: NO!)
- Under  $H_0$ , each person has the same creativity score no matter what treatment.
- Differences in average creativity between repeated samples is just due to randomly assigning each person to each group.
- We can simulate this random mechanism.



## The idea of resampling is to

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- resample (sample from the sample)
- with or without replacement
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- I create different realizations of possible experimental results (if the null hypothesis were actually true).
- I compare many, many resampled experimental results with the observed experimental results I decide if observed result is common or rare to occur by chance.
- If observed data are rare compared to resampled results: the data may point to something interesting (an effect)
- If observed data are common within resampled results: maybe result just occurred by chance (no evidence of an effect)

## Another Sample

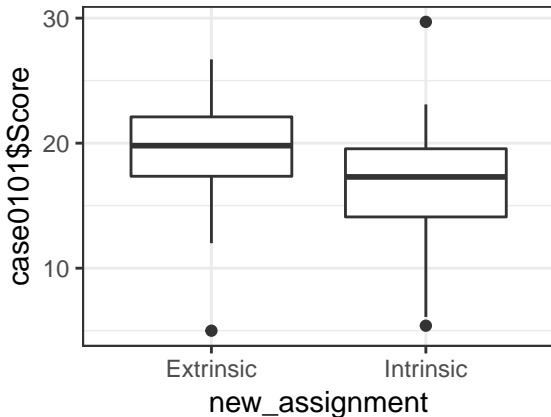
```
new_assignment <- sample(case0101$Treatment)
```

```
new_assignment
```

```
## [1] Extrinsic Intrinsic Intrinsic Intrinsic Intrinsic E  
## [8] Intrinsic Intrinsic Intrinsic Intrinsic Intrinsic I  
## [15] Intrinsic Extrinsic Intrinsic Intrinsic Extrinsic E  
## [22] Extrinsic Extrinsic Intrinsic Intrinsic Extrinsic E  
## [29] Extrinsic Extrinsic Extrinsic Intrinsic Intrinsic E  
## [36] Extrinsic Extrinsic Intrinsic Intrinsic Extrinsic E  
## [43] Intrinsic Extrinsic Extrinsic Extrinsic Intrinsic  
## Levels: Extrinsic Intrinsic
```

## EDA of new sample

```
qplot(new_assignment, case0101$Score, geom = "boxplot")
```



## Another Sample

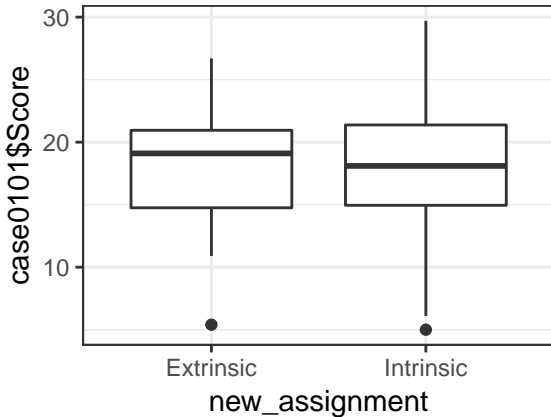
```
new_assignment <- sample(case0101$Treatment)
```

```
new_assignment
```

```
## [1] Intrinsic Extrinsic Intrinsic Extrinsic Extrinsic I
## [8] Intrinsic Intrinsic Intrinsic Extrinsic Intrinsic I
## [15] Extrinsic Intrinsic Intrinsic Intrinsic Intrinsic B
## [22] Extrinsic Extrinsic Extrinsic Extrinsic Intrinsic I
## [29] Extrinsic Extrinsic Extrinsic Extrinsic Extrinsic B
## [36] Extrinsic Intrinsic Intrinsic Intrinsic Extrinsic I
## [43] Extrinsic Intrinsic Intrinsic Extrinsic Intrinsic
## Levels: Extrinsic Intrinsic
```

## EDA of new sample

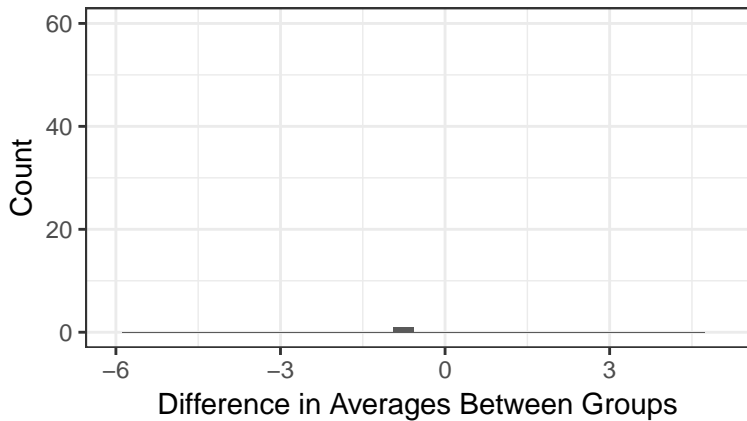
```
qplot(new_assignment, case0101$Score, geom = "boxplot")
```



## Many Samples i

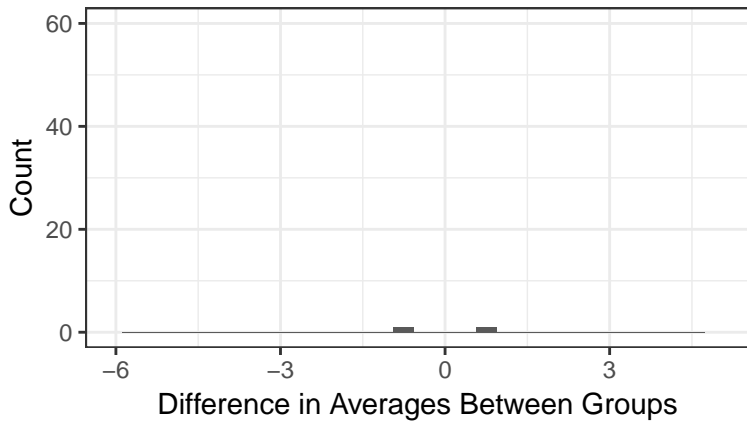
```
set.seed(1)
itermax <- 5000
diffvec <- rep(NA, length = itermax)
for (index in seq_len(itermax)) {
  new_assignment <- sample(case0101$Treatment)
  diffvec[index] <-
    mean(case0101$Score[new_assignment == "Intrinsic"]) -
    mean(case0101$Score[new_assignment == "Extrinsic"])
}
```

## Many Samples ii

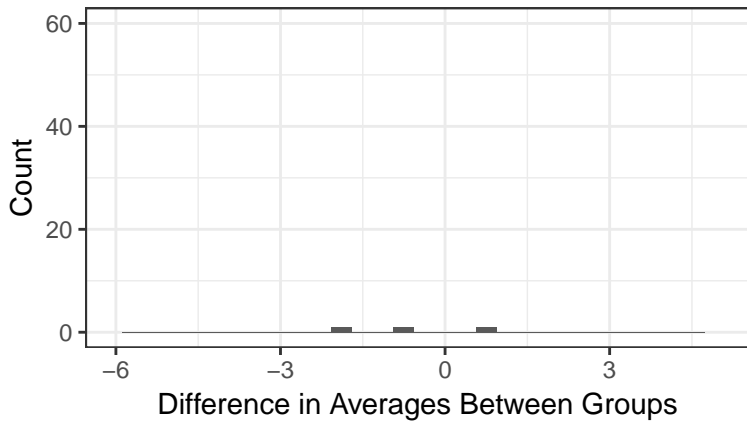




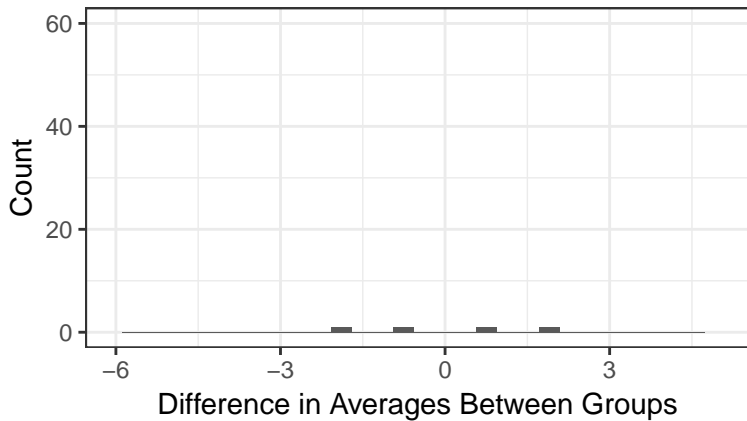
## Many Samples iii



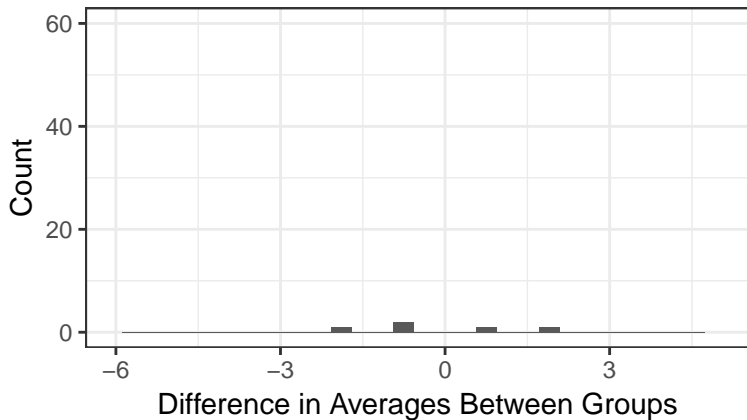
## Many Samples iv



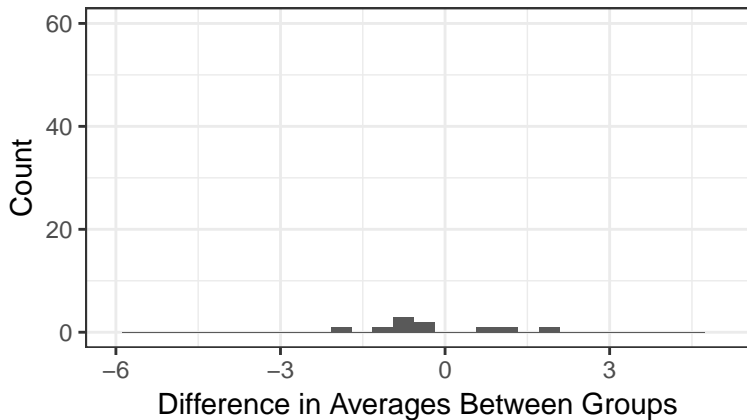
## Many Samples $v$



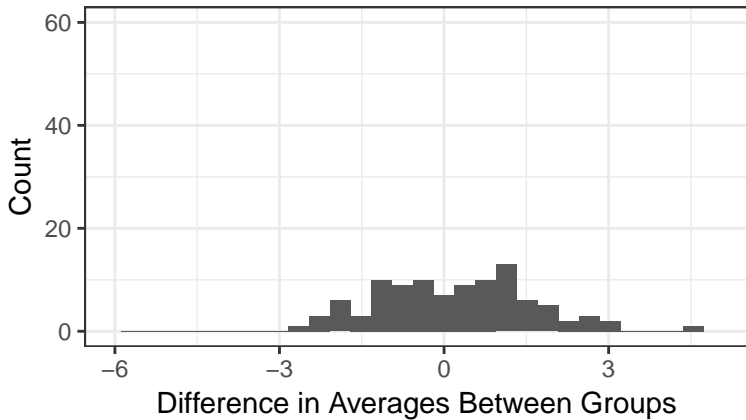
## Many Samples $v_i$



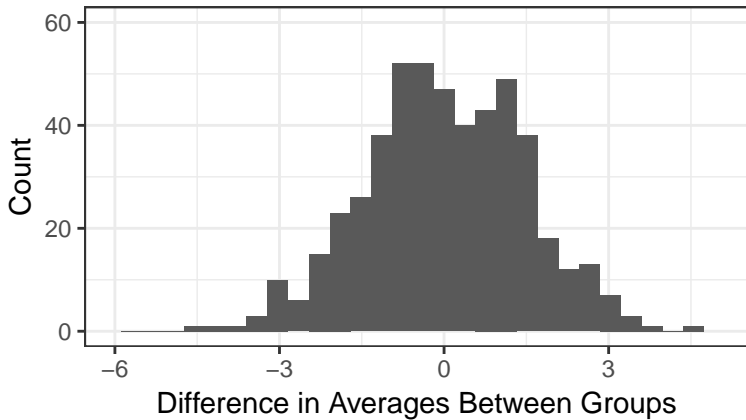
## Many Samples vii



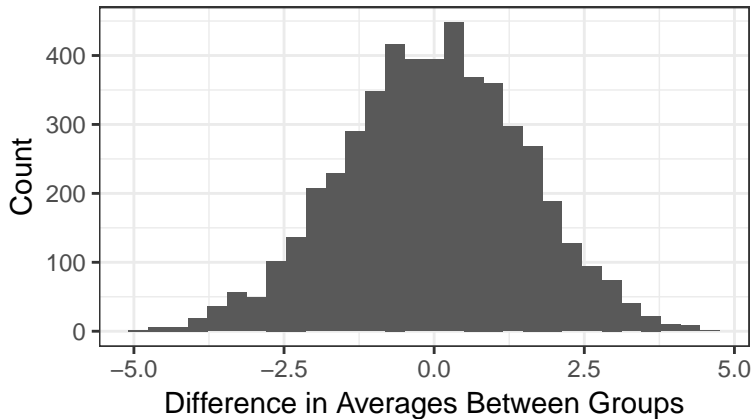
## Many Samples viii



## Many Samples ix

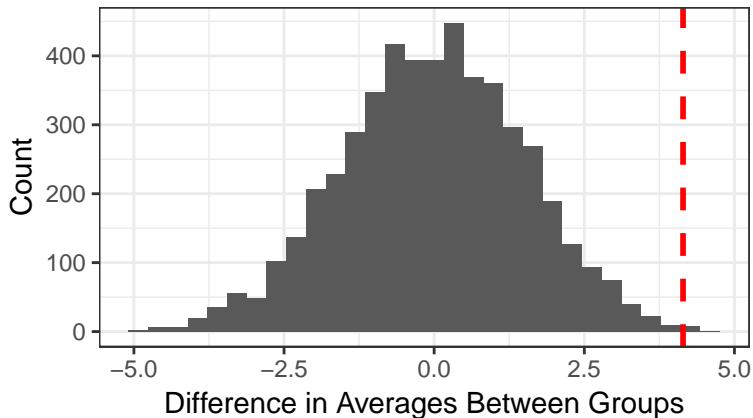


## Many Samples $x$





## Compare to our Observed Difference



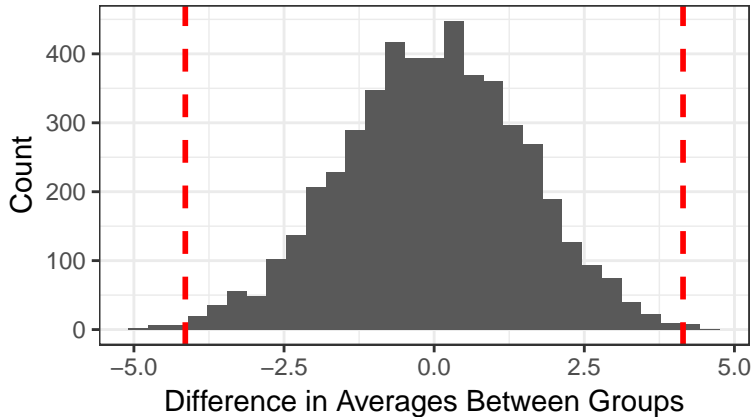
## One sided hypothesis

What proportion of random assignments have a score greater than or equal to the observed score?

```
mean(diffvec > int_mean - ext_mean)
```

```
## [1] 0.0016
```

## Compare to magnitude of difference



## Two sided p-value

What proportion of random assignments have a score as favorable or more favorable to the alternative than our observed score?

```
mean(abs(diffvec) > int_mean - ext_mean)
```

```
## [1] 0.0044
```