03 Probability Review

2018-12-07

Example 1

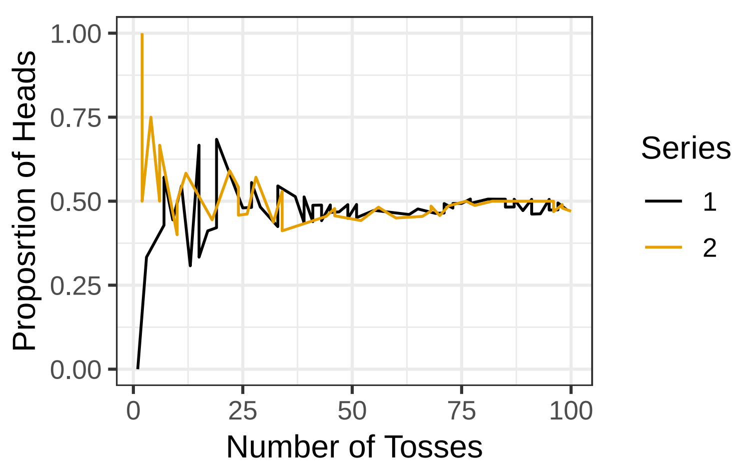
a. Toss a coin and record the result. What is the probability of a heads? Tails?

b. Roll a 6-sided die and record the result. What is the probability of getting a 1? 2? Etc.

c. Toss two coins (or one coin twice). What is the probability of getting two heads?

d. What does it mean that there is a 10% chance of rain today?

A phenomenon is **random** if individual outcomes are uncertain, but there is nonetheless a regular distribution of outcomes in a large number of repetitions. The **probability** of any outcome of a random phenomenon can be defined as the proportion of times the outcome would occur in a very long series of repetitions.



Probability distributions or **models** describe random processes and consist of two parts:

1. a list of all possible outcomes (called the sample space)

2. the probability of each outcome

Example 2

Give the probability distribution for Example 1b.

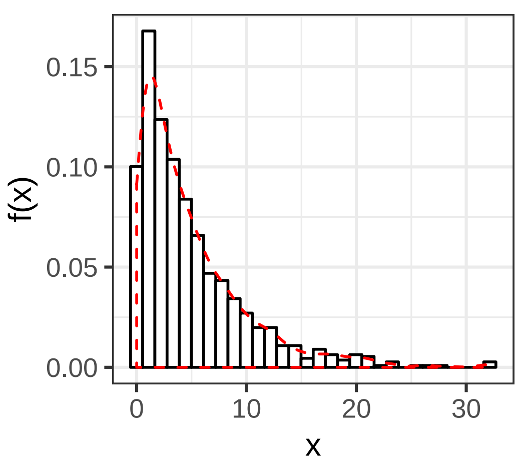
Example 3

Suppose you count the number of heads after two flips of a coin. Find the probability distribution of the number of heads.

Distributions are frequently described by their histogram or density curve.

Example 4

Suppose the probability distribution (model) of X = the number of minutes that a randomly selected student uses his or her cell phone (for any purpose) in a given hour has a population density/histogram given in the figure below. Describe the distribution of X. (Note: X is called a random variable.)



Probability Distributions of a Population

- Distributions describe behavior: the values a random variable can take and how likely it will take on these particular values (the probability of each value).
- Probability histograms or density curves (called pdf or probability distribution function) provide graphical and mathematical descriptions of probability distributions.
- The histogram or density curve can be used to find probabilities that the random variable takes on given values.

Probability Rules - Let A be an event or a random variable

- $0 < P(A) < 1$
- $P(A^c) = 1 - P(A)$ where A^c is the complement of A.
- $P(A \text{ or } B) = P(A) + P(B)$ provided A and B are disjoint. Two events are disjoint if they have no outcomes in common and cannot occur at the same time.

Example 5

The event $A =$ flip exactly two heads is disjoint from the event $B =$ flip exactly two tails.

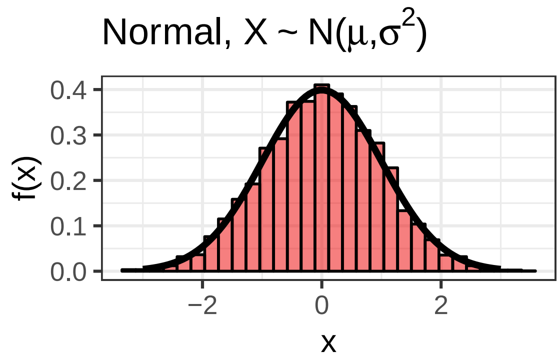
- $P(A \text{ and } B) = P(A)P(B)$ provided A is **independent** of B. Two events are independent if knowing that one event occurs does not change the probability that the other occurs.

Example 6

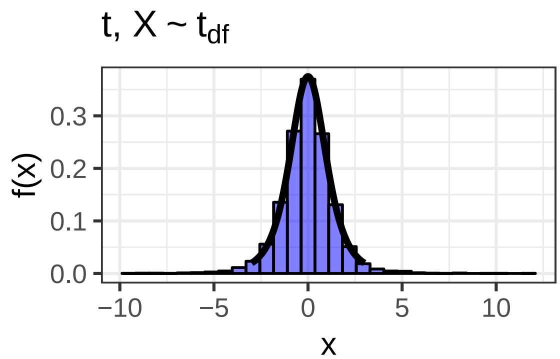
Let T be the event that you get tail on the first toss and H be the event that you get a head on the second toss.

Some Famous Distributions

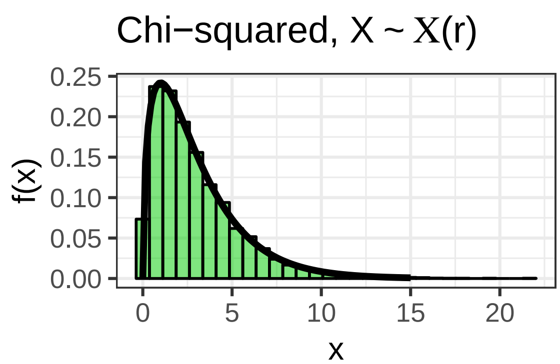
- Normal.
  - `dnorm()`, `rnorm()`, `pnorm()`, `qnorm()`
  - $\mu =$ the mean. The center of the distribution.
  - $\sigma^2 =$ variance. Larger implies more spread out.



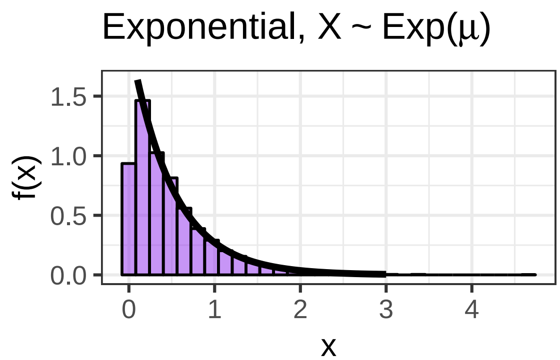
- t
  - `dt()`, `rt()`, `pt()`, `qt()`
  - $\nu = df$ = "Degrees of freedom". Smaller implies more spread out.
  - Mean is always 0.



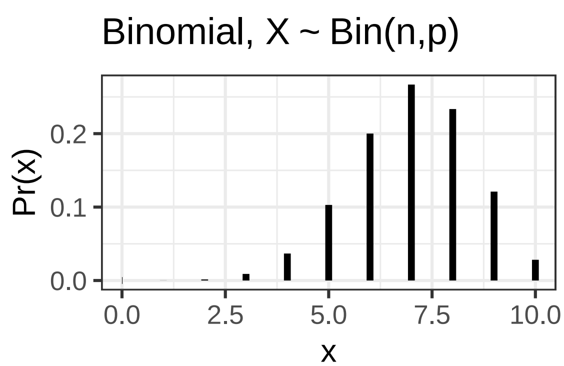
- Chi-squared
  - `dchisq()`, `rchisq()`, `pchisq()`, `qchisq()`.
  - $\nu = df$ = degrees of freedom. Smaller implies more spread out.



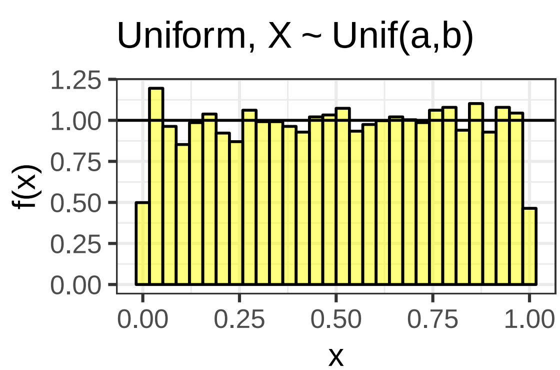
- Exponential
  - `dexp()`, `rexp()`, `pexp()`, `qexp()`
  - λ = "rate parameter".
  - Usually used for questions like "how long before a lightbul fails"



- Binomial
  - `dbinom()`, `rbinom()`, `pbinom()`, `qbinom()`
  - p = success probability
  - n = size
  - Suppose (i) there are n trials, (ii) Each trial results one of two possible outcomes, which we will call "success" and "failure", (iii) The probability of success is p for all trials, and (iv) all trials are independent of each other. Then the number of success is binomially distributed with size n and probability p.

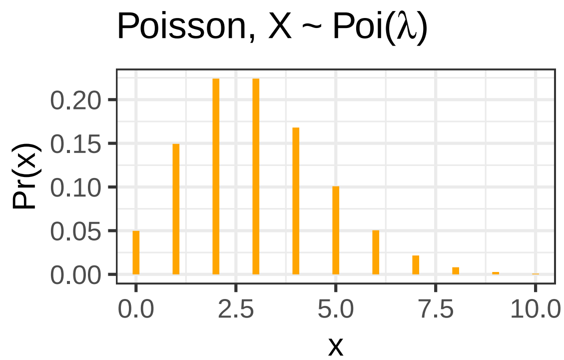


- Uniform
  - `dunif()`, `runif()`, `punif()`, `qunif()`.
  - a = lower bound
  - b = upper bound
  - "Every value has an equal probability"



- Poisson
  - `dpois()`, `rpois()`, `ppois()`, `qpois()`.

- λ = rate
- Has both mean and variance equal to λ.
- Used to answer questions like "how many phone calls will we get in a fixed amount of time."



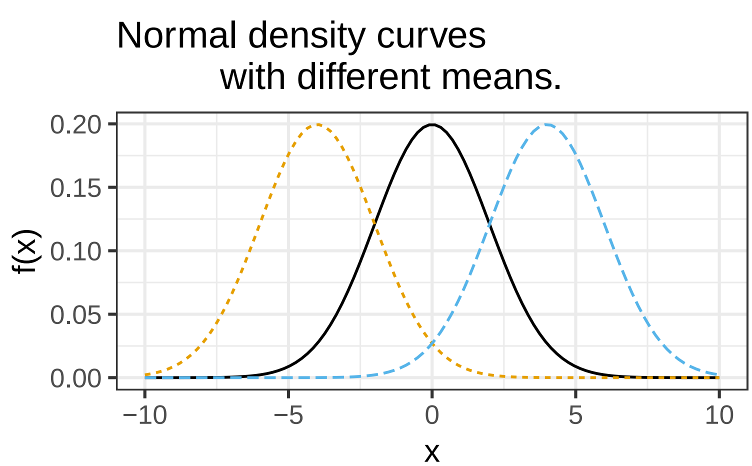
Normal Density Curves:

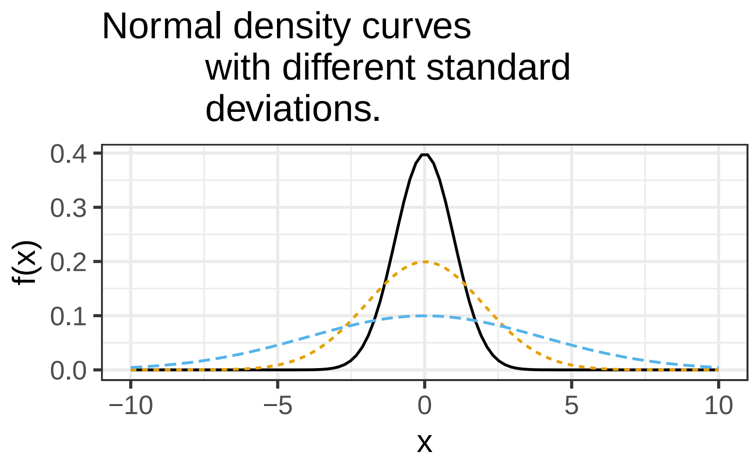
$N(\mu, \sigma^2)$

- For those who are interested, the expression for the normal density curve is:

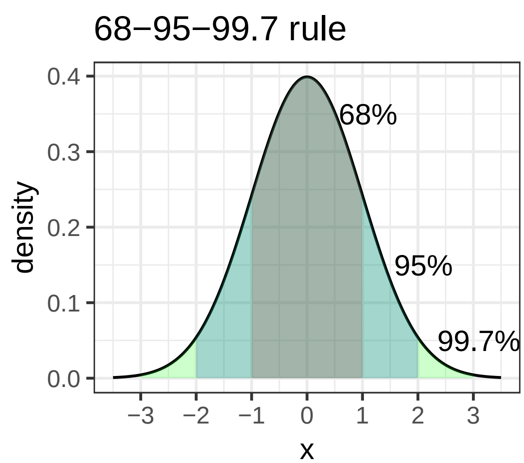
$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Different values of μ shift the curve left or right along the x-axis without changing the shape. Different values of σ stretch or squeeze the curve.





- 68-95-99.7 Rule:
  - Approximately 68% of the observations fall within one σ of μ
  - Approximately 95% of the observations fall within two σ of μ (it's really 1.96 not 2)
  - Approximately 99.7% of the observations fall within three σ of μ.



- Variables which have a $N(\mu, \sigma)$ distribution are often first standardized so that they have a $N(0, 1)$ distribution by subtracting the mean and dividing by the standard deviation:

$$z = \frac{x - \mu}{\sigma}$$

Example

Suppose the calf circumference of all adults has a normal distribution with mean 37 cm and standard deviation of 5 cm, at least approximately. We use the notation

$$X \sim N(\mu, \sigma^2)$$

to denote a normal distribution with mean μ and standard deviation σ (variance σ^2). Here X = calf circumference is in cm, the mean is $\mu = 37$ cm and the standard deviation is $\sigma = 5$ cm.

a. What is the probability that a randomly selected person has a calf circumference less than 37 cm?

b. What proportion has calf circumference greater than 37 cm?

c. What is the probability that a randomly selected person from the population will have a calf circumference between 27 cm and 47 cm?

d. What is the probability that two randomly selected adults from the population will each have a calf circumference greater than 47 cm?

Computing Probabilities in R

In the R workspace for $X \sim N(\mu, \sigma^2)$, to find $Pr(X < a)$ enter: `pnorm(a, $\mu$, $\sigma$)`. Note that in R you must input the standard deviation **not** the variance.

a. Pr(X < 30)

```
pnorm(30, 37, 5)
```

```
## [1] 0.08076
```

b. Pr(30 < X < 35) = Pr(X < 35) - P(X < 30)

```
pnorm(35, 37, 5) - pnorm(30, 37, 5)
```

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## [1] 0.2638
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