Multi-factorial ANOVA

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Objectives

- Brief summary of how we generalize to more than two factors.

Poker Case Study

- Skill: Expert, Average
- Hand: Bad, Neutral, Good
- Limit: Fixed, None
- Cash: Final cash balance

Source: G. Meyer, M. von Meduna, T. Brosowski, T. Hayer (2012). "Is poker a Game of Skill or Chance? A Quasi-Experimental Study," Journal of Gambling Studies, Online First DOI 10.1007/s10899-012-9327-8

Data

```
load("../data/poker_skill.rdata")
head(poker)

## # A tibble: 6 x 4
##   Skill  Hand  Limit  Cash
##   <fct>  <fct> <fct> <dbl>
## 1 Expert Bad   Fixed  4
## 2 Expert Bad   Fixed  5.55
## 3 Expert Bad   Fixed  9.45
## 4 Expert Bad   Fixed  7.19
## 5 Expert Bad   Fixed  5.71
## 6 Expert Bad   Fixed  5.32
```

Full Interaction Model

- $Y_{ijk\ell} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \epsilon_{ijk\ell}$

- μ: baseline mean.

- α_i, β_j, γ_k: main effects

- $(\alpha\beta)_{ij}$, $(\alpha\gamma)_{ik}$, $(\beta\gamma)_{jk}$: two-way interactions.

- $(\alpha\beta\gamma)_{ijk}$: three-way interaction
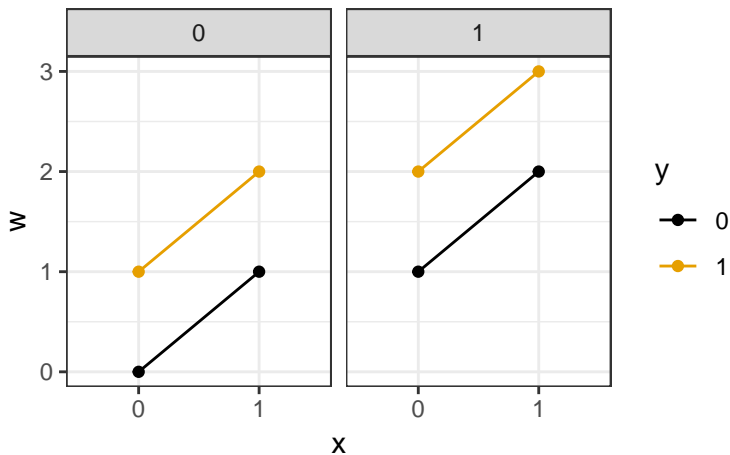
Interaction Plots

- One variable on x-axis

- One variable as grouping variable.

- One variable distinguish facets

Interaction Plots

- Two-way interaction without three-way interaction: the effects change, but the interaction effect is the same between any pair of factors as the level of the third factor changes.

- Three way interaction: the interaction effect changes as we plot at different levels of the third factors.
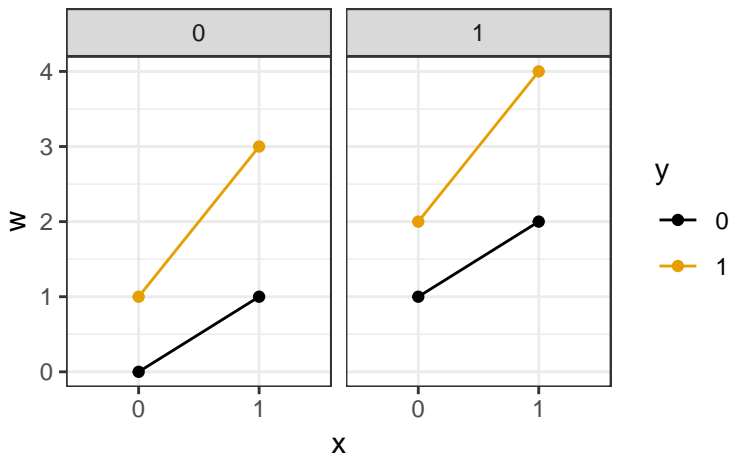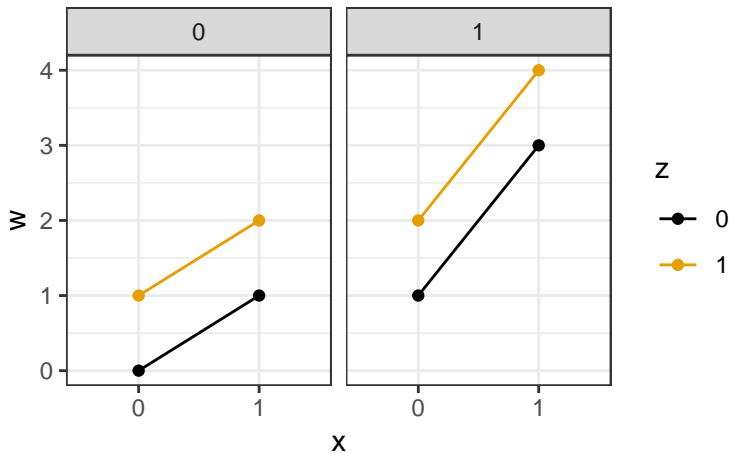
No Interactions

- Facets are *z*

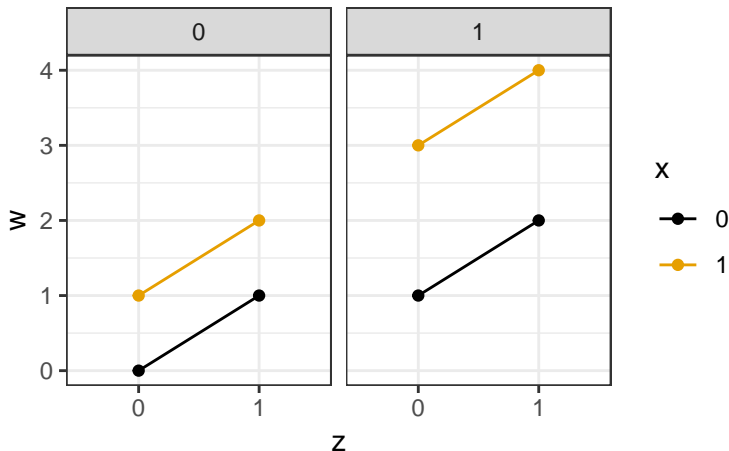# Two-way interactions (x and y only), no three-way interactions
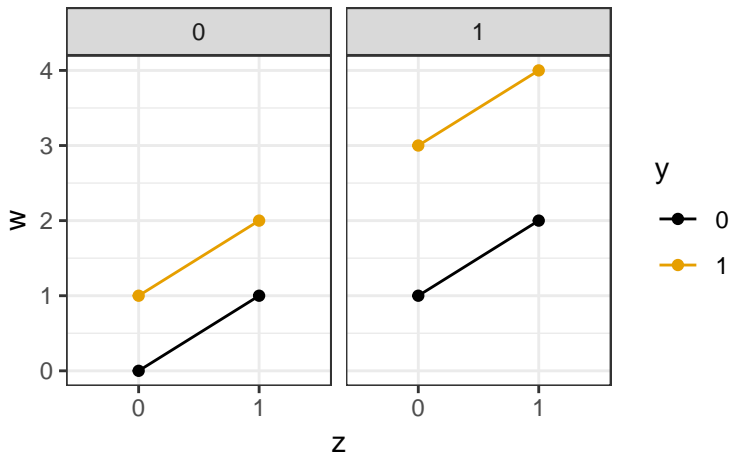
- Facets are z

Two-way interactions (x and y only), no three-way interactions

- Facets are y

Two-way interactions (x and y only), no three-way interactions
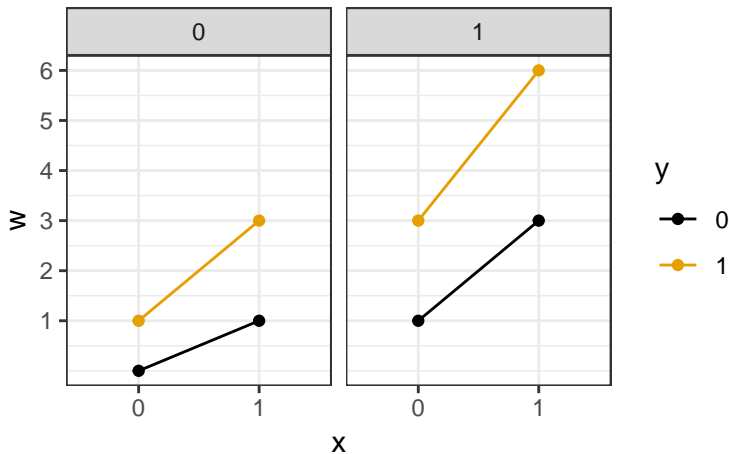
- Facets are y

Two-way interactions (x and y only), no three-way interactions
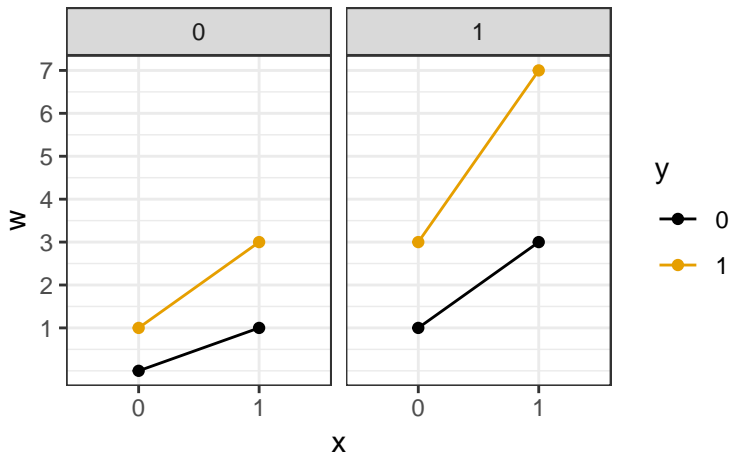
- Facets are x

Two way interactions (between all variables), no three way

- Facets are *z*

Three-way interactions

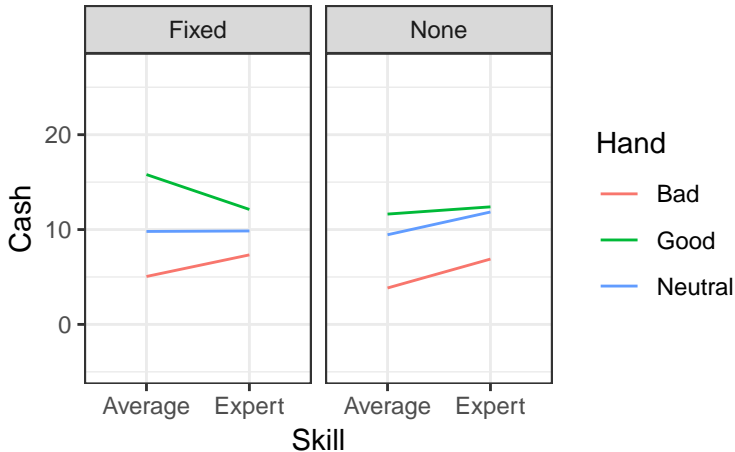- Facets are *z*

Interaction Plots

```
qplot(x = Skill, y = Cash, color = Hand,
      group = Hand, facets = . ~ Limit,
      geom = "blank", data = poker) +
  stat_summary(fun.y = mean, geom = "line")
```

Interaction Plots

```
qplot(x = Hand, y = Cash, color = Skill,
      group = Skill, facets = . ~ Limit,
      geom = "blank", data = poker) +
  stat_summary(fun.y = mean, geom = "line")
```

Interaction Plots

```
qplot(x = Limit, y = Cash, color = Skill,
      group = Skill, facets = . ~ Hand,
      geom = "blank", data = poker) +
  stat_summary(fun.y = mean, geom = "line")
```

Interaction Plots

```
qplot(x = Skill, y = Cash, color = Limit,
      group = Limit, facets = . ~ Hand,
      geom = "blank", data = poker) +
  stat_summary(fun.y = mean, geom = "line")
```

Interaction Plots

```
qplot(x = Hand, y = Cash, color = Limit,
      group = Limit, facets = . ~ Skill,
      geom = "blank", data = poker) +
  stat_summary(fun.y = mean, geom = "line")
```
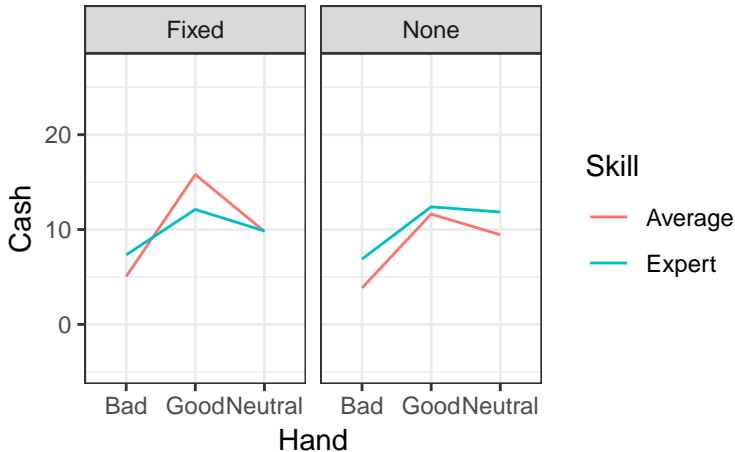
Interaction Plots

```
qplot(x = Limit, y = Cash, color = Hand,
      group = Hand, facets = . ~ Skill,
      geom = "blank", data = poker) +
  stat_summary(fun.y = mean, geom = "line")
```
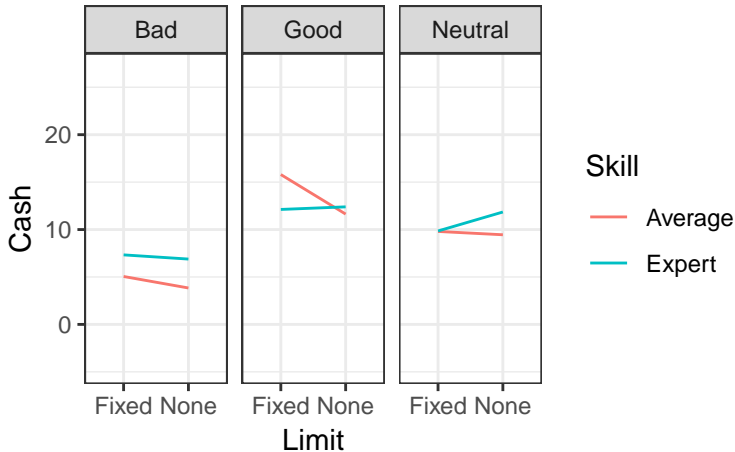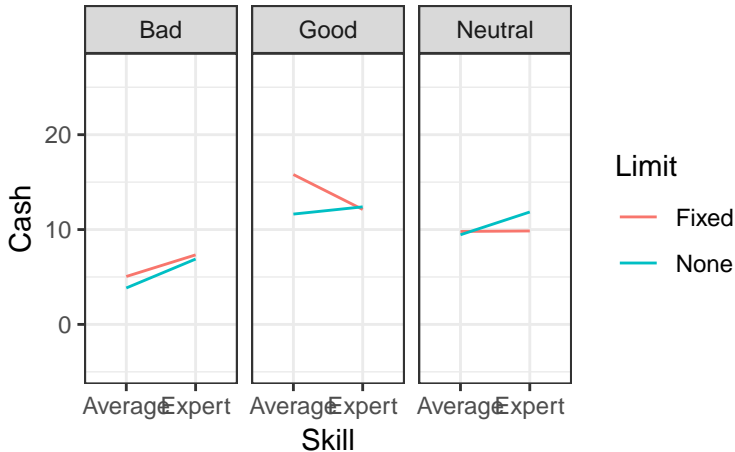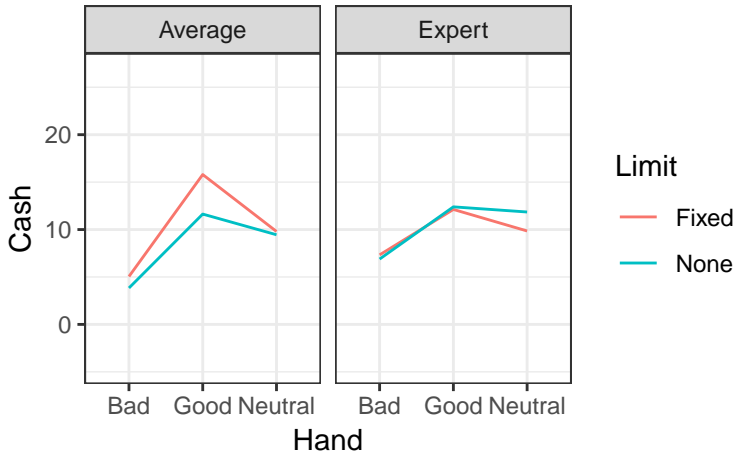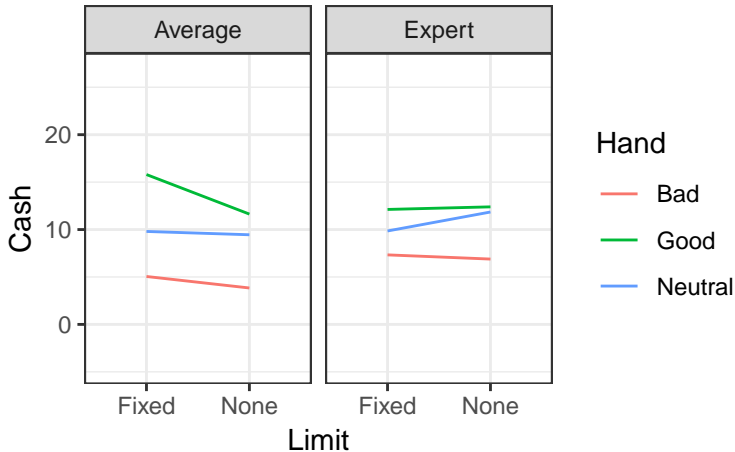
In R

```r
## No evidence of a three-way interaction
aout_cell <- aov(Cash ~ Skill * Hand * Limit, data = poker)
summary(aout_cell)

##                   Df Sum Sq Mean Sq F value Pr(>F)
## Skill              1     49      49    2.84 0.0931
## Hand               2   2647    1323   76.41 <2e-16
## Limit              1     32      32    1.83 0.1773
## Skill:Hand         2    219     110    6.32 0.0021
## Skill:Limit        1    119     119    6.88 0.0092
## Hand:Limit         2     97      49    2.81 0.0619
## Skill:Hand:Limit   2     42      21    1.22 0.2957
## Residuals        288   4987      17
```

In R

```r
aout_sub <- aov(Cash ~ Skill + Hand + Limit +
                Skill * Hand + Skill * Limit,
              data = poker)
summary(aout_sub)
```

```
##               Df Sum Sq Mean Sq F value Pr(>F)
## Skill          1     49      49    2.80 0.0953
## Hand           2   2647    1323   75.36 <2e-16
## Limit          1     32      32    1.80 0.1802
## Skill:Hand     2    219     110    6.24 0.0022
## Skill:Limit    1    119     119    6.78 0.0097
## Residuals    292   5127      18
```
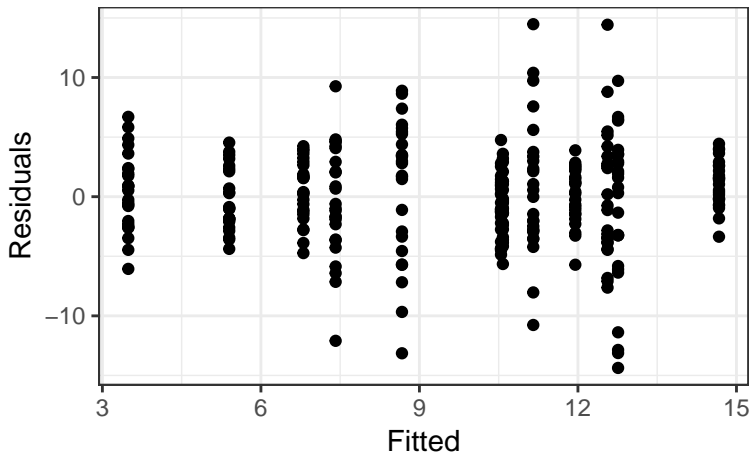
Residual Plots

```
fvec <- fitted(aout_sub)
rvec <- resid(aout_sub)
qplot(fvec, rvec, xlab = "Fitted", ylab = "Residuals")
```

Still interesting

```
t(t(coef(aout_sub)))
```

```
##                          [,1]
## (Intercept)             5.400
## SkillExpert             1.404
## HandGood                9.270
## HandNeutral             5.180
## LimitNone              -1.910
## SkillExpert:HandGood   -4.124
## SkillExpert:HandNeutral -1.440
## SkillExpert:LimitNone   2.520
```

Interpretations

Variable	Estimate
μ	5.400
α_1	0 (by definition)
α_2	1.404
β_1	0 (by definition)
β_2	5.180
β_3	9.270
γ_1	0 (by definition)
γ_2	-1.910
$(\alpha\beta)_{11}$	0 (by definition)
$(\alpha\beta)_{12}$	0 (by definition)
$(\alpha\beta)_{13}$	0 (by definition)
$(\alpha\beta)_{21}$	0 (by definition)
$(\alpha\beta)_{22}$	-1.440
$(\alpha\beta)_{23}$	-4.124
$(\alpha\gamma)_{11}$	0 (by definition)
$(\alpha\gamma)_{12}$	0 (by definition)
$(\alpha\gamma)_{21}$	0 (by definition)
$(\alpha\gamma)_{22}$	2.520

Interpretations

- α for skill level

- β for hand

- γ for limit

- Mean difference between experts with good hands and experts with bad hands is

$$\mu + \alpha_2 + \beta_3 + \gamma_k + (\alpha\beta)_{23} + (\alpha\gamma)_{2k}$$
$$- [\mu + \alpha_2 + \beta_1 + \gamma_k + (\alpha\beta)_{21} + (\alpha\gamma)_{2k}]$$
$$= \mu + \alpha_2 + \beta_3 + \gamma_k + (\alpha\beta)_{23} + (\alpha\gamma)_{2k}$$
$$- [\mu + \alpha_2 + 0 + \gamma_k + 0 + (\alpha\gamma)_{2k}]$$
$$= \beta_3 + (\alpha\beta)_{23}$$
$$= 9.270 - 4.124 = 5.146$$

Interpretations

- α for skill level

- β for hand

- γ for limit

- Mean difference between non-experts with good hands and non-experts with bad hands is

$$\mu + \alpha_1 + \beta_3 + \gamma_k + (\alpha\beta)_{13} + (\alpha\gamma)_{1k}$$
$$- [\mu + \alpha_1 + \beta_1 + \gamma_k + (\alpha\beta)_{11} + (\alpha\gamma)_{1k}]$$
$$= \beta_3$$
$$= 9.270$$

- Experts benefit less from a quality hand?

Interpretations

- Similarly ideas for limit.

- SkillExpert:LimitNone > 0 indicates that experts benefit **more** from not having a limit.