Two-way ANOVA Main Effects

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Objectives

- Testing for Main Effects
- Chapter 13 in the book.

Pygmalion Effect Case Study

- Pygmalion Effect: high expectations of a supervisor translate into improved performance of subordinate.
- A company of soldiers contains three platoons each.
- Within each company, one platoon was randomly selected to be the "Pygmalion platoon."
- The platoon leader in the Pygmalion platoon was told by the army psychologist that his platoon was predicted to be superior.
- At end of basic training, all soldiers in each platoon were given a skill test.
- Data consist of average scores for each platoon.

The data

```
library(Sleuth3)
data("case1302")
head(case1302)
```

```
##   Company     Treat Score
## 1      C1 Pygmalion  80.0
## 2      C1   Control  63.2
## 3      C1   Control  69.2
## 4      C2 Pygmalion  83.9
## 5      C2   Control  63.1
## 6      C2   Control  81.5
```

Two-way ANOVA model: The additive model

- Model: $Y_{ijk} = \mu + \alpha_i + \beta_j + \epsilon_{ijk}$
- Y_{ijk}: Value of observational unit j of group i of the first categorical variable and group j of the second categorical variable.

Two-way ANOVA model: The additive model

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- Y_{ijk}: Value of observational unit j of group i of the first categorical variable and group j of the second categorical variable.
- μ: baseline value.
- α_i: Additive effect of being in group i in categorical variable 1.
- β_j: Additive effect of being in group j in categorical variable 2.

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- μ: baseline value.
- α_i: Additive effect of being in group i in categorical variable 1.
- β_j: Additive effect of being in group j in categorical variable 2.
- ϵ_{ijk}: Individual-specific noise for observational unit k of group i of the first categorical variable and group j of the second categorical variable. Assumed to have mean 0 and variance σ^2.
- σ^2 is assumed to be the **same** for all observational units of all groups

Main Effects

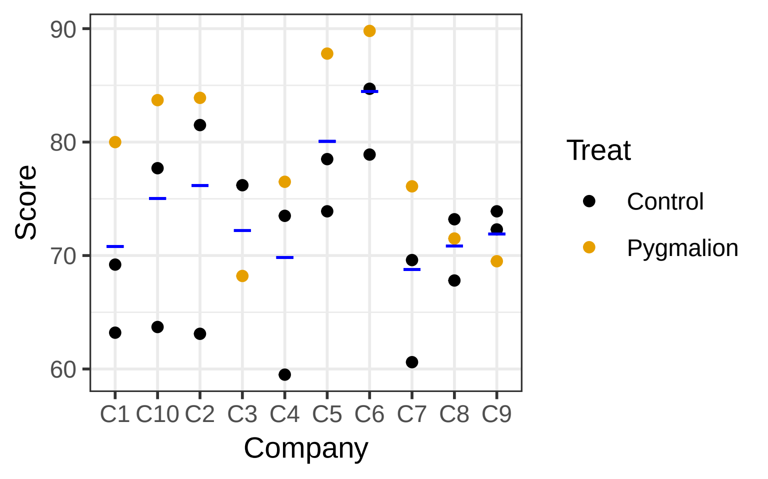
- The α_i's are called the "main effects" for company (as opposed to interaction effects).
- The β_j's are called the "main effects" for treatment.
- We are not directly interested in the α_i's. The reasearchers just thought that there would be a lot of variability between companies and so wanted to control for this.
- We want to know if the treatment is effective.
- $H_0 : \beta_i = 0$ for all i.
- $H_A : \beta_i \neq 0$ for some i.

Our Specific Case

- $J = 2$ since there are two groups (Pygmalion and Control)
- $\beta_1 = 0$ by assumption.
- β_2 is the treatment effect.
- $H_0 : \beta_2 = 0$
- $H_A : \beta_2 \neq 0$

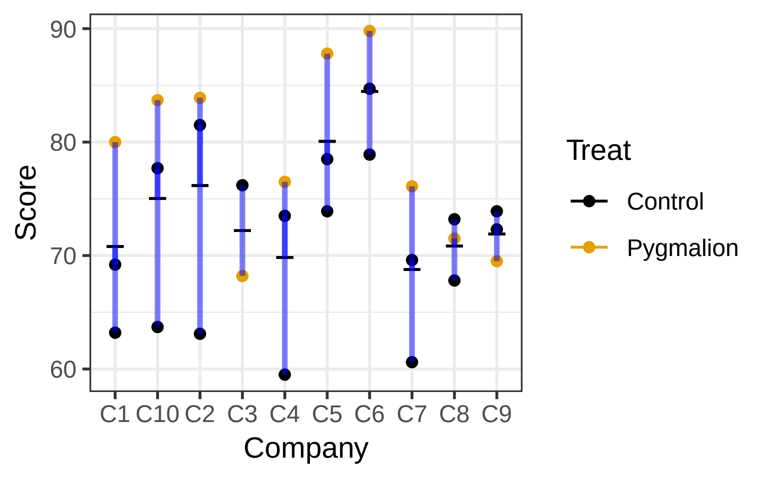
Reduced (Company Only) Model

- Mean estimates



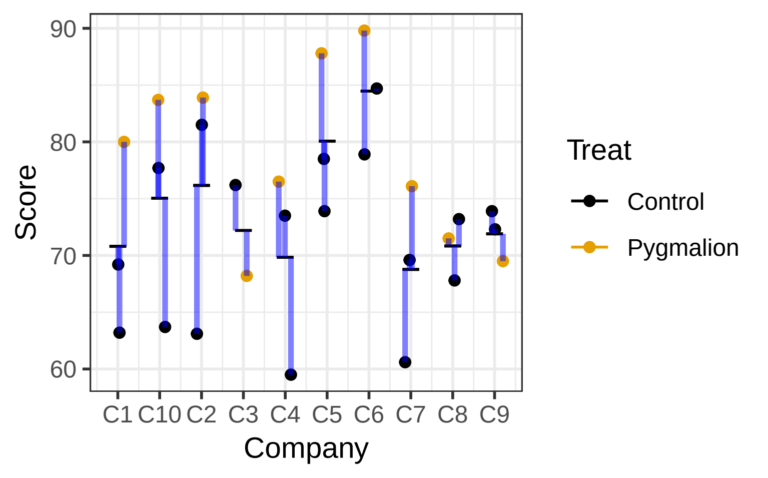
Reduced (Company Only) Model

- Find the Residuals



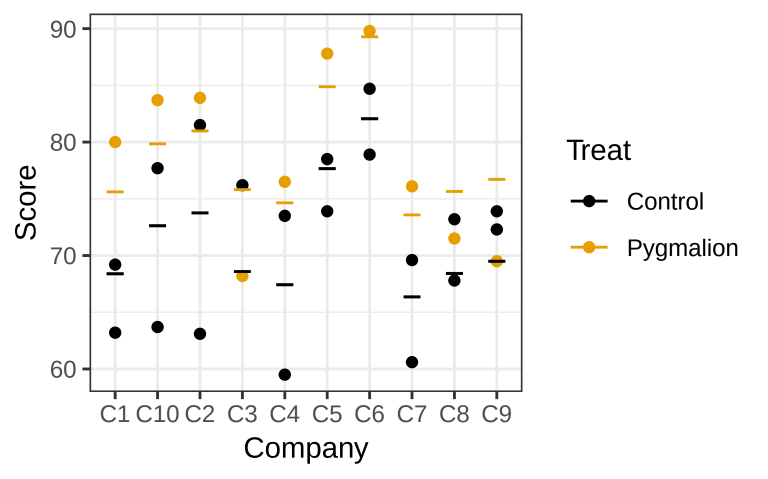
Reduced (Company Only) Model

- Find the Residuals (jittered for easier viewing)



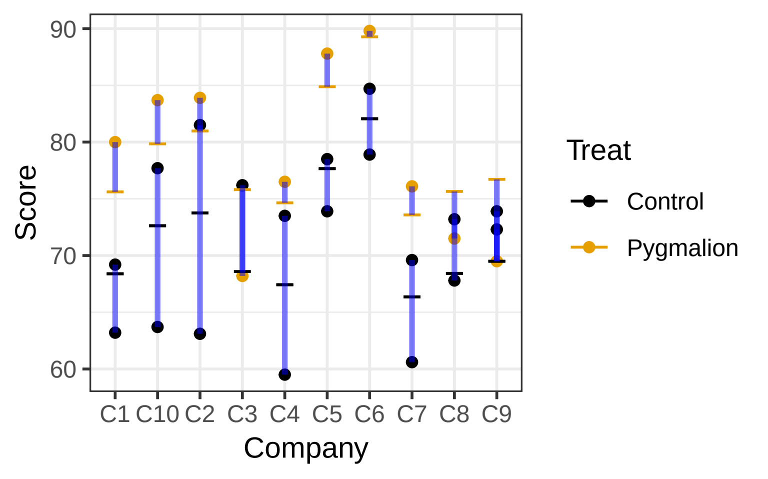
Full (Additive) Model

- Mean Estimates



Full (Additive) Model

- Find the Residuals



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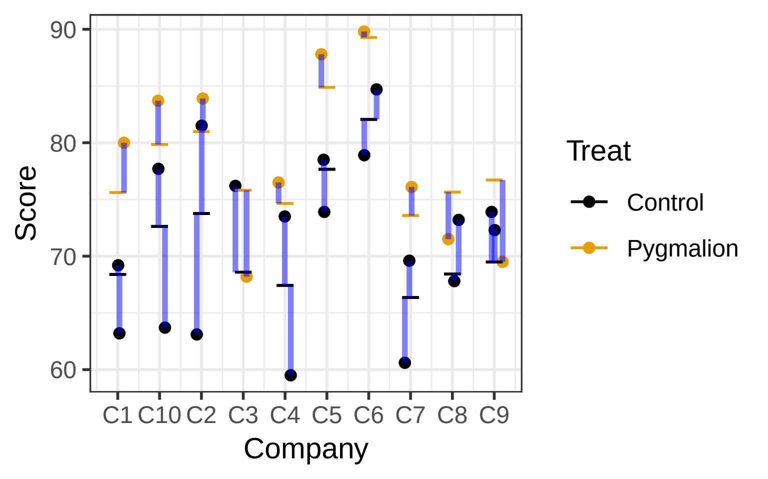

Sum of Squared Residuals

- $RSS_{full} = 778.5039$
- $df_{full} = n - I - J + 1 =$ 29 - 10 -2 $+ 1 = 18$

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- $df_{reduced} = n - I =$ 29 - 10 = 19

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- $ESS = RSS_{reduced} - RSS_{reduced} = 338.8827$
- $df_{extra} = df_{reduced} - df_{full} = 1$

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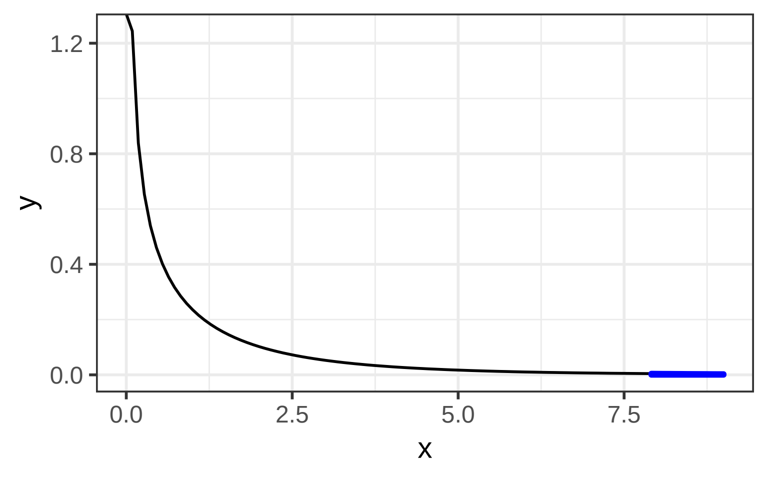
- $ESS = RSS_{reduced} - RSS_{reduced} = 338.8827$
- $df_{extra} = df_{reduced} - df_{full} = 1$
- F-statistic $= \frac{ESS/df_{extra}}{RSS_{full}/df_{full}} = 7.8354.$

Compare observed 7.8354 to a $F_{1,18}$ Distribution



Compare observed 7.8354 to a $F_{1,18}$ Distribution

```r
pf(7.84, df1 = 1, df2 = 18, lower.tail = FALSE)
```

```
## [1] 0.01184
```

- Moderate evidence to suggest a treatment effect.

In R

```
aout_add <- aov(Score ~ Company + Treat, data = case1302)
anova(aout_add)
```

```
## Analysis of Variance Table
##
## Response: Score
##           Df Sum Sq Mean Sq F value Pr(>F)
## Company    9    671      75    1.72  0.156
## Treat      1    339     339    7.84  0.012
## Residuals 18    779      43
```

Get out Coefficient Estimates

```
t(t(coef(aout_add)))
```

```
##                        [,1]
## (Intercept)      68.39316
## CompanyC10        4.23333
## CompanyC2         5.36667
## CompanyC3         0.19658
## CompanyC4        -0.96667
## CompanyC5         9.26667
## CompanyC6        13.66667
## CompanyC7        -2.03333
## CompanyC8         0.03333
## CompanyC9         1.10000
## TreatPygmalion    7.22051
```

Get Confidence Intervals of Effects

```
confint(aout_add)
```

```
##                   2.5 % 97.5 %
## (Intercept)      60.214 76.572
## CompanyC10       -7.048 15.515
## CompanyC2        -5.915 16.648
## CompanyC3       -12.449 12.842
## CompanyC4       -12.248 10.315
## CompanyC5        -2.015 20.548
## CompanyC6         2.385 24.948
## CompanyC7       -13.315  9.248
## CompanyC8       -11.248 11.315
## CompanyC9       -10.181 12.381
## TreatPygmalion    1.801 12.640
```